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A MIXED FINITE DIFFERENCE-FINITE ELEMENT APPROACH TO SIMULATE UNCONFINED FLOW IN THE CRESCENTINO AREA

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ABSTRACT

A small unconfined basin in the Crescentino area (northwestern Italy) is pumped by a number of wells owned partly by a private firm and partly by the public aqueduct. The private pumpage is a matter of concern in that it might undermine the water supply for civil use. An evaluation of the system resources is made by applying a numerical model. The flow field is simulated with a regional model whose results are improved with a local refined model in the vicinity of the single pumping wells. The former model is based on the Dupuit assumptions and relies on a finite difference technique. The latter is based on the equations of flow with a free boundary and uses an iterative finite element approach. Its most valuable application rests on the interpretation of the pumping tests which enable us to derive an accurate estimate of the aquifer constants. Real field simulations show that, if the big concern withdraws its declared quantity (0.5 m^3/s), the related effect on the pumping rate from the public wells is small. A prospective future demand for an additional 1 m^3/s may be satisfied by drilling 5 new wells.

INTRODUCTION

In recent years numerical models have been used to an increasing degree to simulate the behavior of natural aquifer systems. The facilities offered by processor technology as well as the widespread use of high-speed digital computers provide a distinct possibility to realistically handle large and complex hydrologic basins. The starting point is given by the basic equations controlling groundwater flow along with their initial and boundary conditions. Several solution approaches have been devised but the numerical techniques offer the most powerful and flexible tool of analysis.

In the present work a small unconfined basin in the Crescentino area (north-western Italy) is being simulated by use of two separate models. The need for two models springs from the different scale of the physical event under examination wherein a dinstinction must be made between large-scale flow over the entire system and local seepage in the vicinity of the single pumping wells. The first approach, that could be labelled regional, relies on the Dupuit assumption. It discards the vertical component of flow and provides the average horizontal flow field on a x-y plane. The second approach is based on theory of flow with a free boundary and is axi-symmetric on a r-z cross section. Both models can be used either separately or jontly depending on the response required. In particular the local model is of great help in determining the aquifer constants through appropriate pumping tests whereas the regional model is suited to give an estimate of the overall water resources from the system as well as its potential future supply under a variety of possible pumping plans.

The non-linear Dupuit-Boussinesq equation is solved with an implicit finite difference technique. The local flow field around the abstraction wells is determined by an iterative finite element approach. The formation characteristics are assumed to be averagely uniform. The thickness is derived from the stratigraphy and the aquifer constants are evaluated through the analysis of pumping data. System boundaries are given by two natural streams and an artificial canal. North-eastward there are no important sources and the hence the unit has its theoretical boundary set infinity in this direction.

A short survey of the sparse available records is provided first. Next the numerical models are described and the related solution techniques are presented. The local model is applied to interpret the pumping tests and to determine the formation parameters. Finally some results from real field simulations are presented together with a prediction of future water supply in connection with a realistic consumption plan.

HYDROGEOLOGY OF CRESCENTINO AREA

The area we are concerned with lies on the intersection between the Dora river, the Po river and the Canale Cavour (Figure 1). Two main pumping places, denoted on the map by the symbols M and F and owned by the public aqueduct and by a big private firm, respectively, exploit the unconfined subsurface resources there. The sediments from which withdrawals are taken are unconsolidated coarse sands and gravels of the Quaternary age. The stratigraphy, reconstructed in a NW-SE cross-section (line a-a of Figure 1) down to a depth of 160 m, is shown in Figure 2. The relevant data, coming from several sparse boreholes, are fewer in the lower 100 meters and therefore some extrapolations in the geology proved necessary. In the entire reconstruction the accurate drilling analysis performed by Sacco (1924) at Cascina Giarrea as early as 1924 has been of paramount importance. Looking at Figure 2 and bearing in mind also the indications from other wells located at some distance from the vertical cross section represented in the given stratigraphy we can recognize the presence of a phreatic formation in the upper 50 m. Very coarse sediments are prevailing throughout with few thin interbedded silts and clays. The underlying sands turn out to be much finer. These confining beds can be regarded altogether as the first aquitard down to a depth of 74 m where coarse sands and gravels appear again. The first artesian aquifer is located in the interval 74-77 m. Its hydraulic head raises above the ground surface according to Sacco's (1924) survey and its potential productivity is high. The litostratigsa-hic reconstruction below this aquifer is a little more difficult due to the sparse and sometimes unreliable information in our possession.

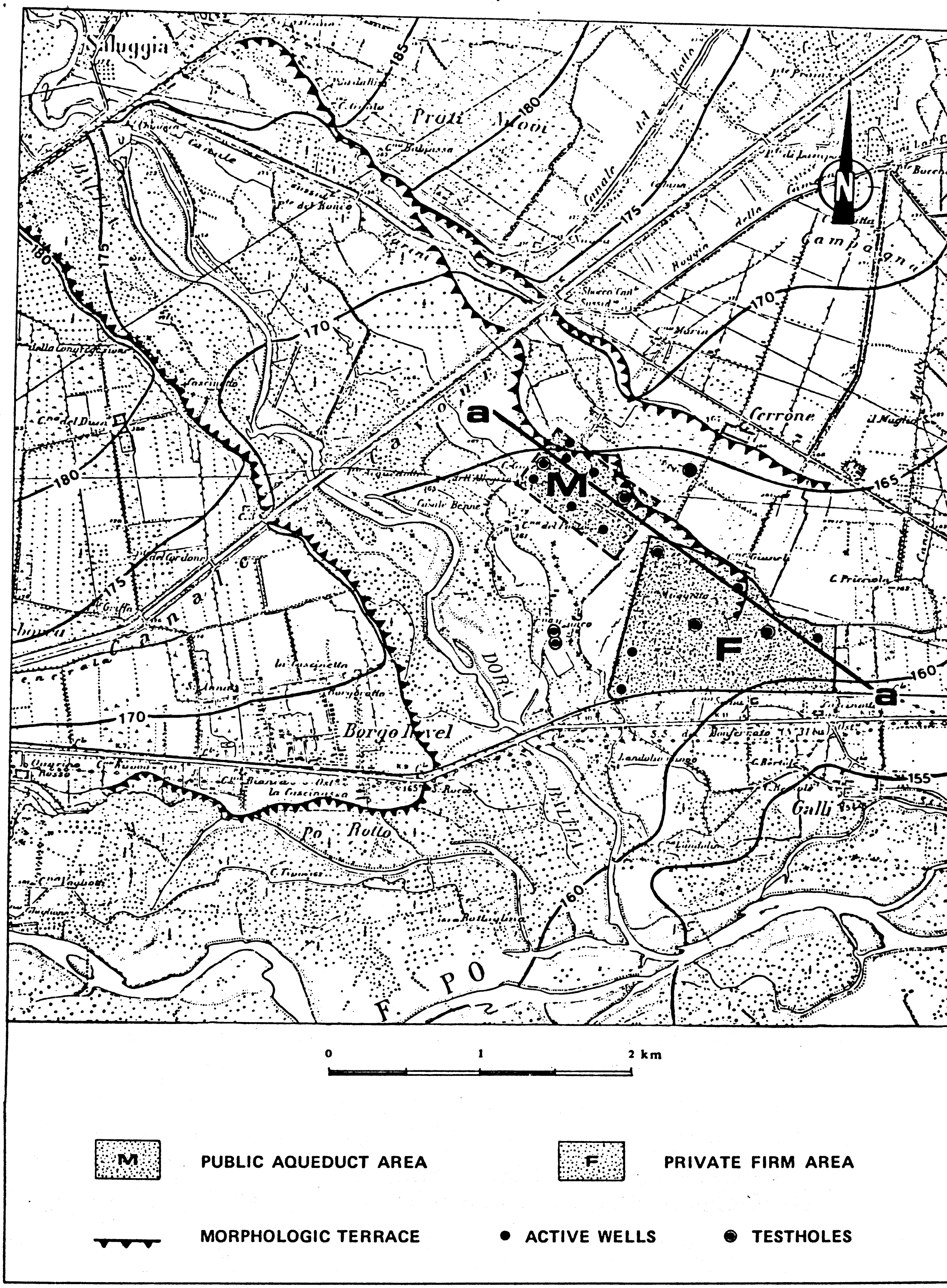

Figure 1. Map of the Crescentino area with the location of the pumping sites.

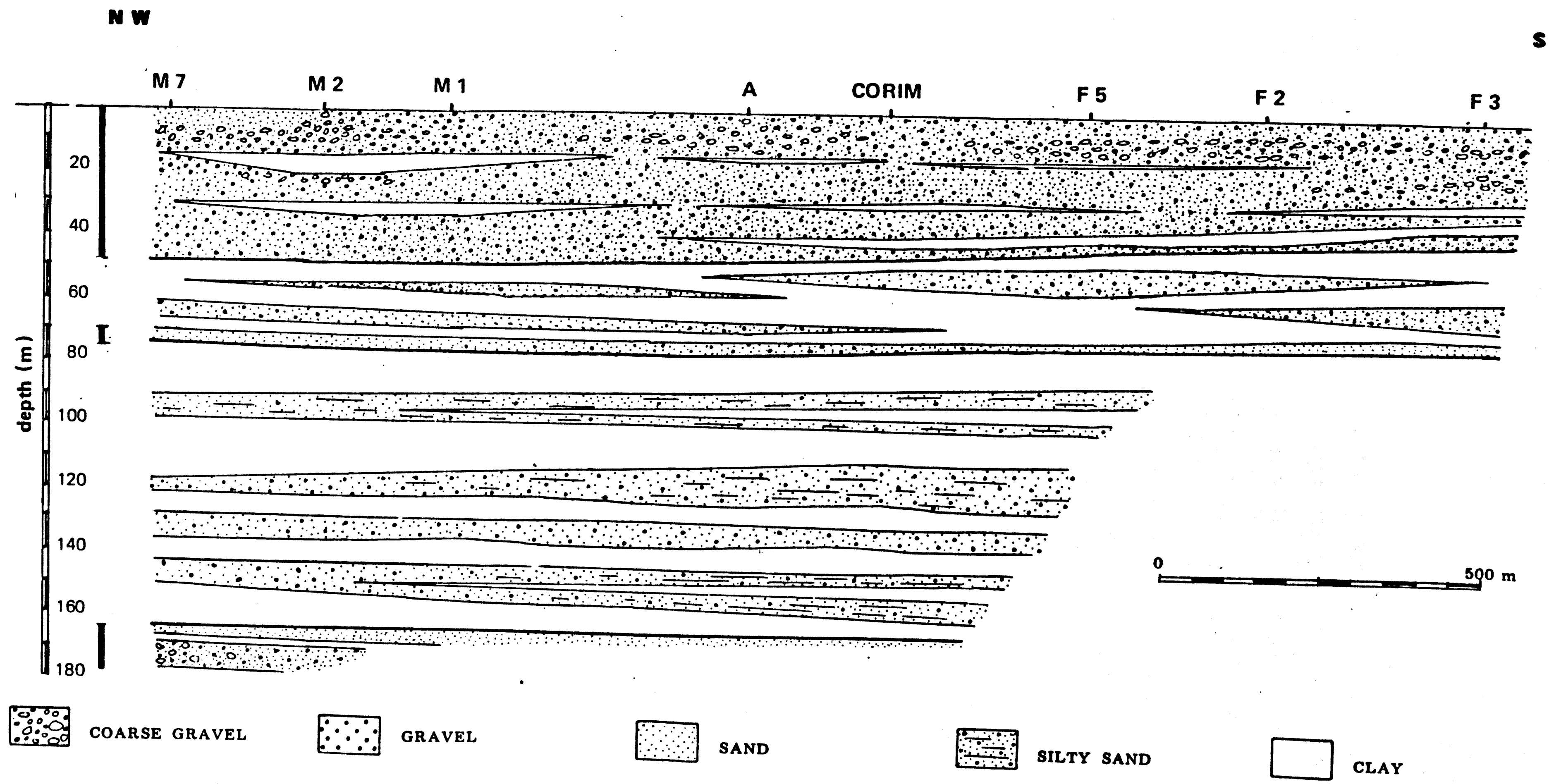

Figure 2. NW-SE litostratigraphic section of the Crescentino area. The location of the main aquifers is indicated.

However we can recognize on the whole a massive low-permeable unit whose thikness is approximately 90 m. It overlies the second confined aquifer. Since pumpage is restricted to the upper sediments a more detailed description beneath the first artesian formation is not really necessary. The topography shows (see Figure 1) that the maximum slope of the ground surface is 0.3% and therefore the aquifers can be taken as horizontal in the simulations ahead. Actually the slight incline causes a steady natural flow prior to pumping as is evidenced by Figure 3 where the steady equipotential lines are drawn on a horizontal map. The streamlines roughly parallel the average axis of the Dora river. The slope of the water table is slightly more pronounced than the ground surface slope. The aquifer recharge boundaries are given by both the Dora stream and the Canale Cavour. It must be noted that even the Po river might contribute some water supply if an intensive pumpage should occur from wells F.

HYDROLOGIC MODEL

Bidimensional horizontal flow.

The regional bidimensional model is based on the Dupuit assumption. Hence it assumes that: a) the vertical flow is negligible; b) the slope of the water table is small. These conditions are well satisfied within the aquifer, except for a limited portion of porous medium around each single pumping well. The transient seepage is governed by the well-known Dupuit-Boussinesq equation that can be written down in the following form:

$$\frac{k}{2}\left(\frac{\partial h^2}{\partial x^2}+\frac{\partial h^2}{\partial y^2}\right)=n_e\frac{\partial h}{\partial t}+W \qquad (1)$$

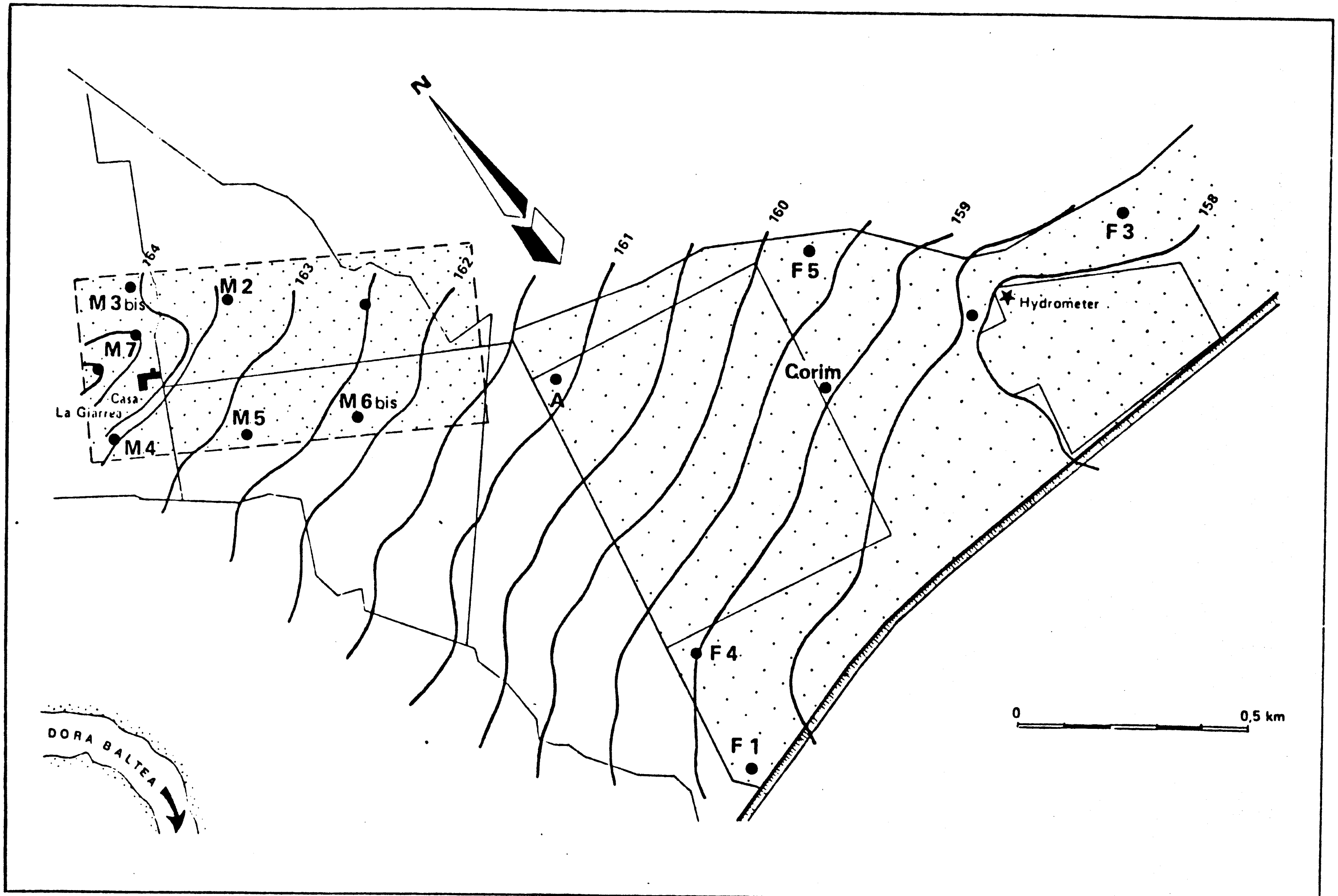

Figure 3. Equipotential lines of the Crescentino unconfined aquifer prior to pumpage.

where h(x,y,t) is the water table elevation above the aquifer bottom, k and n_e are the hydraulic conductivity and the specific yield (or effective porosity), respectively, and W is the sink term. Due to h^2, eq (1) is non-linear. In steady conditions (1) turns into:

$$\frac{K}{2}\left(\frac{\partial h^2}{\partial x^2} - \frac{\partial h^2}{\partial y^2}\right) = W \qquad (2)$$

which is linear in h^2. Boundary conditions require that h be prescribed along the Dora and the Po river and the Canale Cavour. In the NE direction the boundary is set at infinity (in practice in such a position that it gives negligible contribution to the overall material balance). Eq. (1) is solved with the implicit scheme suggested by Crank-Nicolson. This scheme is accurate and unconditionally stable. If the nodal spacings are uniform, discretization of (1) leads to:

$$\frac{1}{2}\left(\Delta_x^2 + \Delta_y^2\right) v_{i,j}^{t+\Delta t} + \frac{1}{2}\left(\Delta_x^2 + \Delta_y^2\right) v_{i,j}^{t} - \frac{2W_{i,j}}{K} =$$

$$= \frac{n_e}{K\Delta t}\,\frac{v_{i,j}^{t+\Delta t} - v_{i,j}^{t}}{\sqrt{v_{i,j}^{t+\Delta t/2}}} \qquad (3)$$

where i and j identify the node lying on the intersection between the i-th row and the j-th column of the network, $v=h^2$, Δt is the time step and:

$$\Delta_x^2 v_{i,j} = \frac{v_{i,j+1} - 2v_{i,j} + v_{i,j-1}}{(\Delta x)^2}$$

$$\Delta_y^2 v_{i,j} = \frac{v_{i+1,j} - 2v_{i,j} + v_{i-1,j}}{(\Delta y)^2}$$

In (3) the quantity $v_{i,j}^{t+\Delta t/2}$ is estimated a priori by using Douglas' (1962) method and applying (3) at time $t+\Delta t/2$. The value $v_{i,j}^{t+\Delta t/4}$ is set directly equal to $v_{i,j}^{t}$. By assembling all eqs. (3) over each node we arrive at a linear pentadiagonal system whose solution is obtained by the ADI method (see Peaceman and Rachford, 1955).

Steady seepage is simulated by solving numerically eq. (2). In order to be able to analyse the aquifer behavior with many simultaneously active wells, a variable nodal spacing must be used. The finite difference scheme by Forsythe and Wasow (1960) yields:

$$\left(\Delta_x^2 + \Delta_y^2\right) v_{i,j} - \frac{2W_{i,j}}{K} = 0 \qquad (4)$$

where Δ_x^2 and Δ_y^2 have here the following expression:

$$\Delta_x^2 v_{i,j} = \frac{2}{\Delta x_{j-1}\,\Delta x_j\,(\Delta x_{j-1} + \Delta x_j)} \left[v_{i,j-1}\,\Delta x_j + v_{i,j}\left(-\Delta x_{j-1} - \Delta x_j\right) + v_{i,j+1}\,\Delta x_{j-1} \right]$$

$$\Delta_y^2 v_{i,j} = \frac{2}{\Delta y_{i-1}\,\Delta y_i\,(\Delta y_{i-1} + \Delta y_i)} \left[v_{i-1,j}\,\Delta y_i + v_{i,j}\left(-\Delta y_{i-1} - \Delta y_i\right) + v_{i+1,j}\,\Delta y_{i-1} \right]$$

Eqs. (4) form a linear system which is solved following the direct approach by Schecter (1960).

Axi-symmetrix flow with a free boundary

Near a pumping well flow is predominantly axi-symmetric. Vertical drainage is significant and the initial water table shape changes appreciably with time. The initial boundary value problem is described by the set of equations (Polubarinova-Kochina, 1962):

$$\frac{K_r}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \phi}{\partial r}\right)+K_z\frac{\partial^2\phi}{\partial z^2}=S_s\frac{\partial \phi}{\partial t} \qquad 0\leqslant z\leqslant h(r,t) \qquad (5)$$

$$\phi(r,z,0)=h(r,0)=b \qquad (5a)$$

$$\phi(R,z,t)=h(R,t) \qquad (5b)$$

$$\frac{\partial \phi}{\partial z}(r,0,t)=0 \qquad (5c)$$

$$\begin{cases} K_r n_r\dfrac{\partial \phi}{\partial r}+K_z n_z\dfrac{\partial \phi}{\partial z}=\left(I-n_e\dfrac{\partial h}{\partial t}\right)n_z & (6a)\\ \phi[r,h(r,t),t]=h(r,t) & (6b)\end{cases} \quad \text{on the free surface}$$

$$\phi(r_i,z,t)=h_i(t) \qquad 0\leqslant z\leqslant h_i(t) \qquad (7a)$$

$$\phi(r_i,z,t)=z \qquad h_i(t)\leqslant z\leqslant BC \qquad (7b)$$

In the previous equations $\phi(r,z,t)$ is the hydraulic potential, b is the aquifer thickness, K_r and K_z are the coefficients of the anisotropic permeability, $S_s=S/b$ where S is the elastic storage, I is the net vertical infiltration rate, n_r and n_z are the direction cosines of the outer normal to the phreatic surface and the remaining symbols are defined in Figure 4. In (7a) and (7b) the water height h_i in the well is unknown and therefore we need an additional relationship between $h_i(t)$ and the pumping rate Q(t). This equation is given by (see also Figure 4):

$$Q(t)=Q_e(t)-\pi(r_i^2-r_w^2)\frac{dh_i}{dt} \qquad (8)$$

where $Q_e(t)$ has the expression:

$$Q_e(t)=2\pi k_r r_i \left[\int_0^{AC}\frac{\partial\phi}{\partial r}dz\right]_{r=r_i} \qquad (9)$$

In (8) r_w is the radius of the production pipe. The value of the outer radius R is taken to be equal to a distance from the well where the vertical seepage becomes small. The correponding free surface elevation h(R,t) is provided by the regional bidimensional model. If the well is partially penetrating or is totally penetrating but partially screened the integration in eq. (9) is extended only to the well portion where the intakes are located. This will really be the case when pumping tests are simulated. Since eq.(6a) is non-linear, the overall problem represented by eqs. (5) through (9) is a non linear one. The solution is achieved by the finite element approach. The difficulty resulting from the non-linearity has been overcome by iteration. The linearized algebraic systems are solved with the over-relaxation technique using an empirical estimate of the optimum acceleration factor. An exten

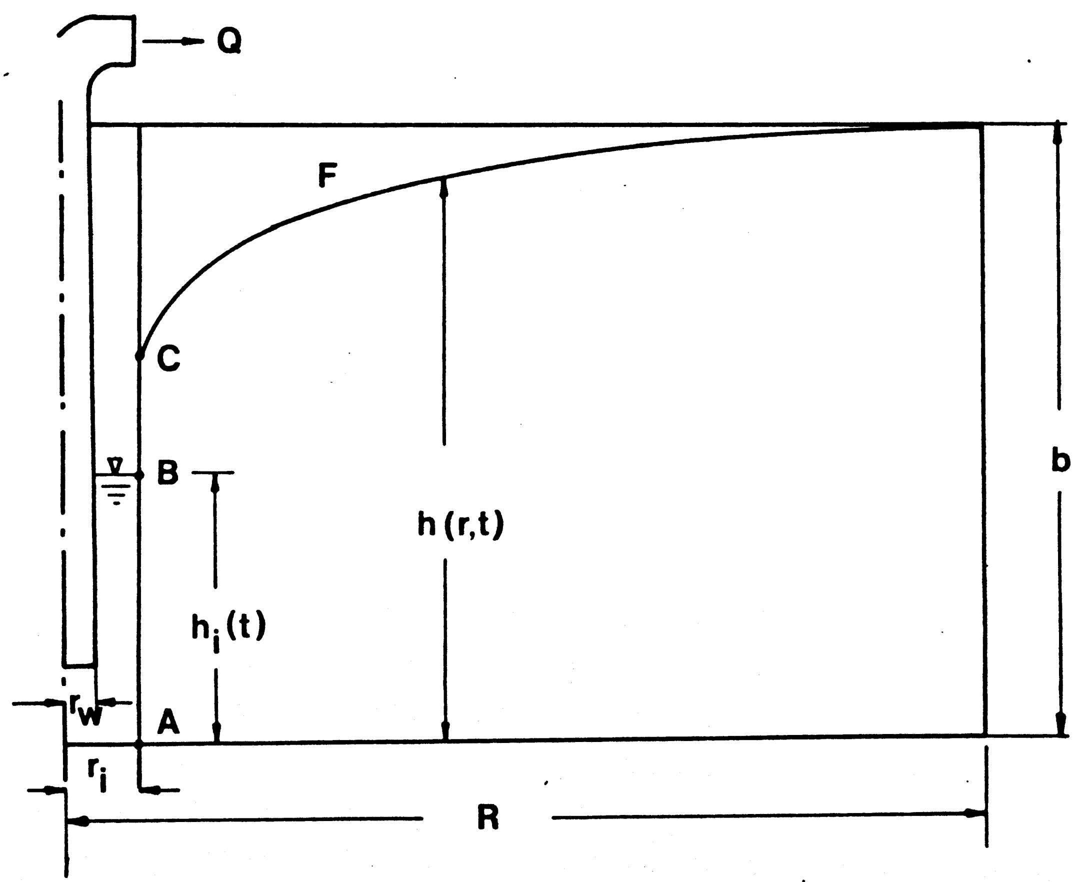

Figure 4. Schematic diagram of an unconfined aquifer with a pumping well.

sive discussion of the whole method would require a long time. The reader is referred to our recent paper (Gambolati, 1976) where the numerical and the computational aspects of the above approach are described in detail.

Limitations of the coupled model.

The local axi-symmetric model takes the water table elevation at some distance R from the well as an input from the regional bidimensional model. The R value must be selected in such a way that the vertical flow is negligibly small for $r=R$, i.e. $\phi(R,t)$ is constant with z. Our experience indicates that this condition is satisfied when $R \simeq b$. As an example we can look at Figure 5 where the correct free surface profile and the profile given by the Dupuit solution are compared. The results refer to an aquifer with R=1669 m and have been derived for the most unfavourable case, i.e. for the case in which the well is pumped at its maximum rate ($h_i = 0$, steady conditions). The inner radius r_i is equal to 0.4 m which is in keeping with the dimensions of both set of wells M and F. Note that the two curves are practically coincident for $r > b = 45$ m thus indicating that the vertical seepage is small beyond $r=b$. This conclusion applies also when a single well is tapping a non-circular aquifer, provided the hole is not too close to a boundary. However if we are concerned with many wells the coupling of the two models requires that interference between every two adjacent wells be small within a circle of radius equal to b. In other words the radial effect around a pumping well must propagate to a distance equal to at least the aquifer thickness. As a major consequence it turns out that the smallest admissible distance between two wells must be of the order of $5 \div 6$ b to avoid a distortion of the cone of depression inside $r=b$. Looking at the map of Figure 1 we can conclude that this limitation does not concern us here.

PUMPING TESTS ANALYSIS

The most valuable application of the local model is the interpretation of the pumping tests. The field trials were performed by the CORIM company in a

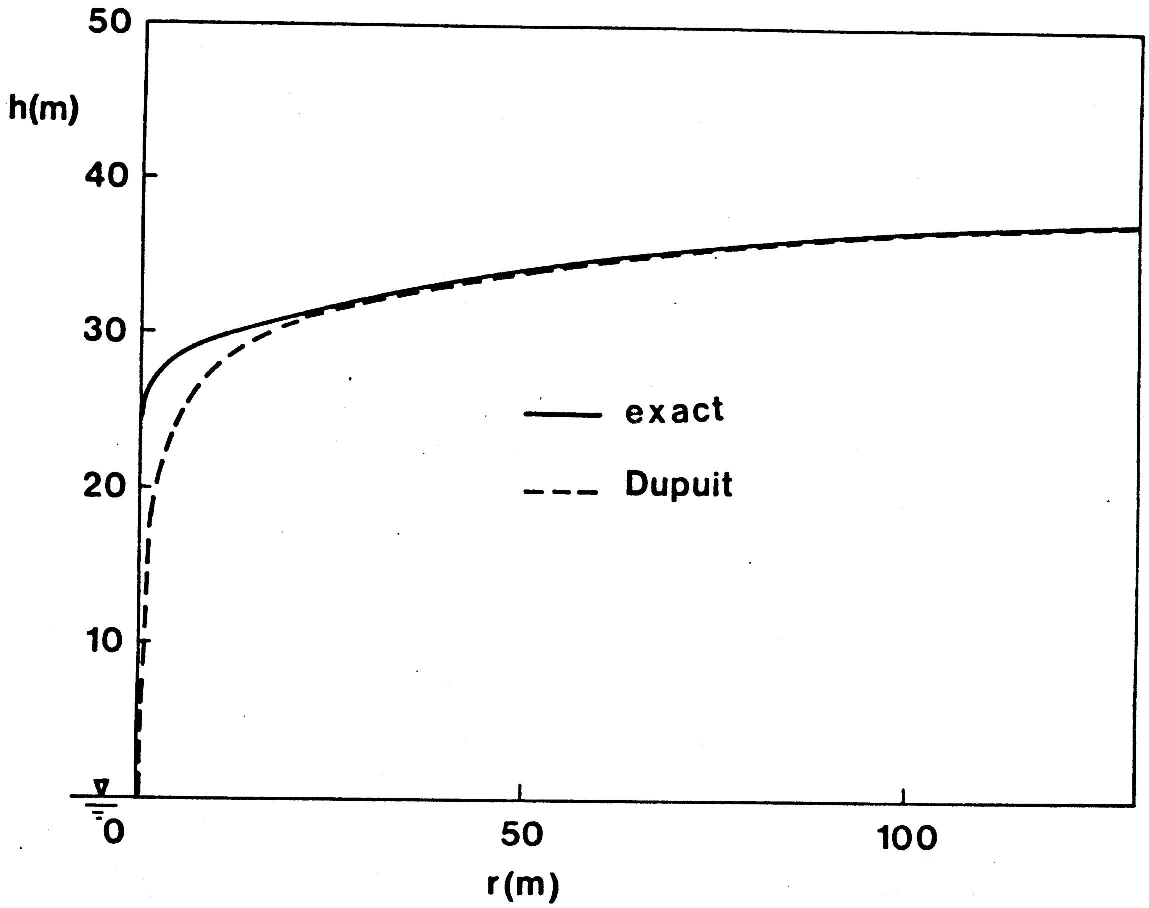

Figure 5. Comparison between the exact and the Dupuit solution for steady flow to a gravity well. The aquifer thickness is set equal to 45 m.

testhole shaped as is shown in Figure 6. Note that 6 piezometers were also arranged on a radial line from the hole. The depth of these observation wells was such as to intercept the position of the water table only. During the trials however the measurements recorded in the piezometers were so modest (few centimeters) that they could not be used in the subsequent simulations. The reasons for this behavior have been investigated theoretically with the model. The outcome is shown in Figure 7 which gives various water table profiles in a circular aquifer for steady conditions and constant discharge rate. R is 130 m and b is the same as that of the Crescentino aquifer. The drawdown in the well has been set equal to 3 m, i.e. equal to the maximum decline of head registered during the tests. Curve 4 refers to a fully penetrating well totally screened with $K_r = K_z = 0.44 \cdot 10^{-3}$ m/s (this numeral will be shown to be the estimated formation permeability). The related Q turns out to be 0.053 m^3/s for an r_i equal to that of the CORIM borehole. If we reduce the screen length as was done in the actual CORIM drilling (see Figure 6) and keep $b-h_i = 3$m, the water table raises as is indicated by profile 2 and simultaneously Q diminishes to 0.0286 m^3/s. With this same Q and a complete intake the free surface would be lowered to curve 3 (the resulting well drawdown would be 1.7m). Finally if we assume that $k_z = 0.1\, k_r$, again with $b-h_i = 3$ and the real position and size of the screen, the water table turns out to be provided by curve 1, i.e. the free surface fall is in this case minimal (Q becomes 0.025 m^3/s). From the previous discussion it is apparent that the upper lining of the CORIM borehole has hampered significantly the decline of the water table and that this effect has been emphasized by a likely anisotropy. A more satisfactory response might have been provided by: a) increasing the pumping rate; b) extending the duration of trials; c) drilling deeper observation wells. Unfortunately these remedies were not used due to deficiencies in both the financial support and the field assistance.

Our analysis is therefore restricted to the testhole drawdown. By the model described earlier we have simulated the decline of head as was recorded in the well during the pumping stage. Excluding the unsaturated zone above the phreatic surface and considering the presence of tiny silty and clayey lenses irregularly scattered within the aquifer, the effective unit thickness b was

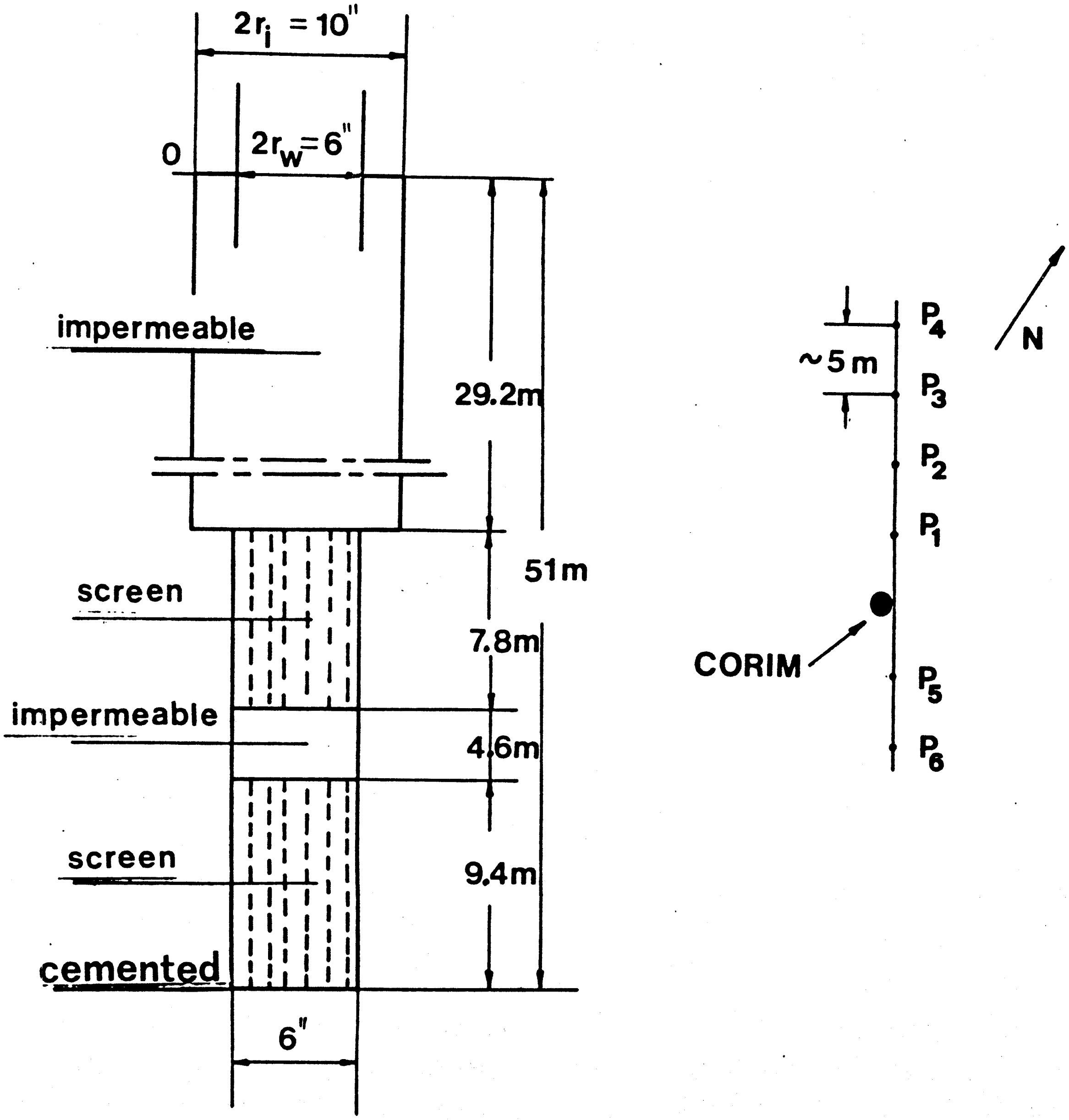

Figure 6. Structure of the Corim borehole. The location of the piezometers is indicated.

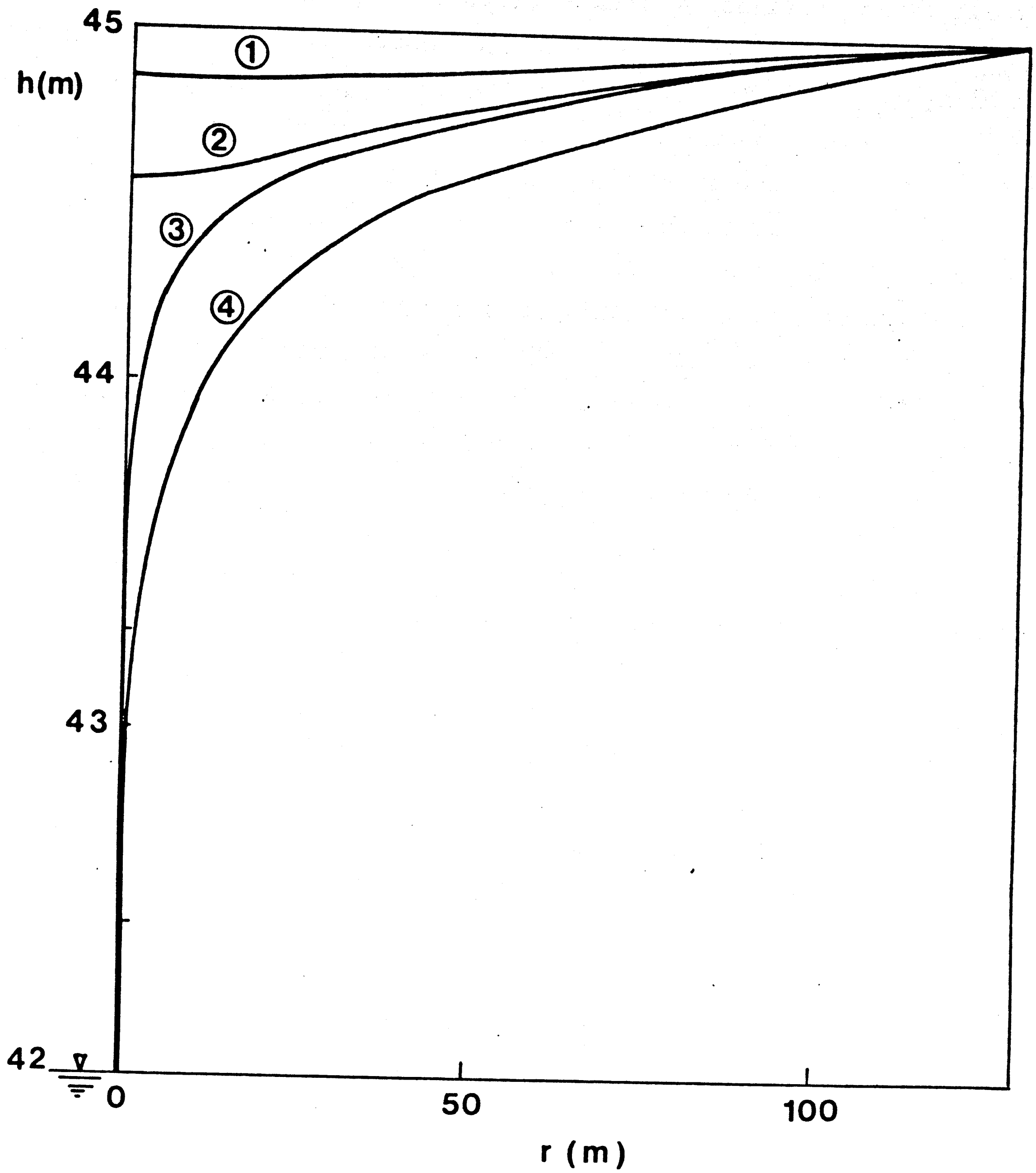

Figure 7. Steady free surface profiles in a circular aquifer under various assumptions.

estimated to be 45 m. The well storage has duly been taken into account. The test data are shown in Table 1. Note that in the piezometers the water level is almost stationary, as has already been pointed out. The formation constants giving the best fit between the theoretical and the observed records, have been found by trial and error. Figure 8 shows a comparison between the observed drawdowns and the outcome from the model obtained with $k_r = k_z = 0.44 \cdot 10^{-3}$ m/s and $n_e = 0.3$. The profile represents the best match we have been able to get by changing the couple of parameters k_r and n_e. It is worth stressing that n_e for coarse sands and gravels varies in a rather restricted range (Bear, 1972, p. 486) and its influence on the solution is relatively small whereas the potential k - variability is much higher. An accurate estimate of the hydraulic conductivity is therefore very important. Due to the accuracy of our model, the above formation parameters are expected to be quite good, at least for the aquifer portion surrounding the test hole. This conclusion is also warranted by the fact that well losses were negligible (unconfined well of large diameter in relation to discharge, see Castany, 1968).

The axi-symmetric model has served another useful purpose as well, i.e. to check the magnitude of leakage from the underlying artesian aquifer. To do so we added the confining beds to the unconfined unit, as is shown in Figure 9 and solved the governing equation in a two-layered system. We had no data regarding the aquitard permeability but from geology it is reasonable to assume that its vertical hydraulic conductivity is at least two order of magnitude smaller that that of the overlying sediments. We assumed a constant head in the first artesian aquifer and found a contribution from the lower boundary which did never exceed 2% of the total steady pumpage.

It may concluded that leakage from below is negligible and hence the unconfined aquifer bottom can be taken as impermeable.

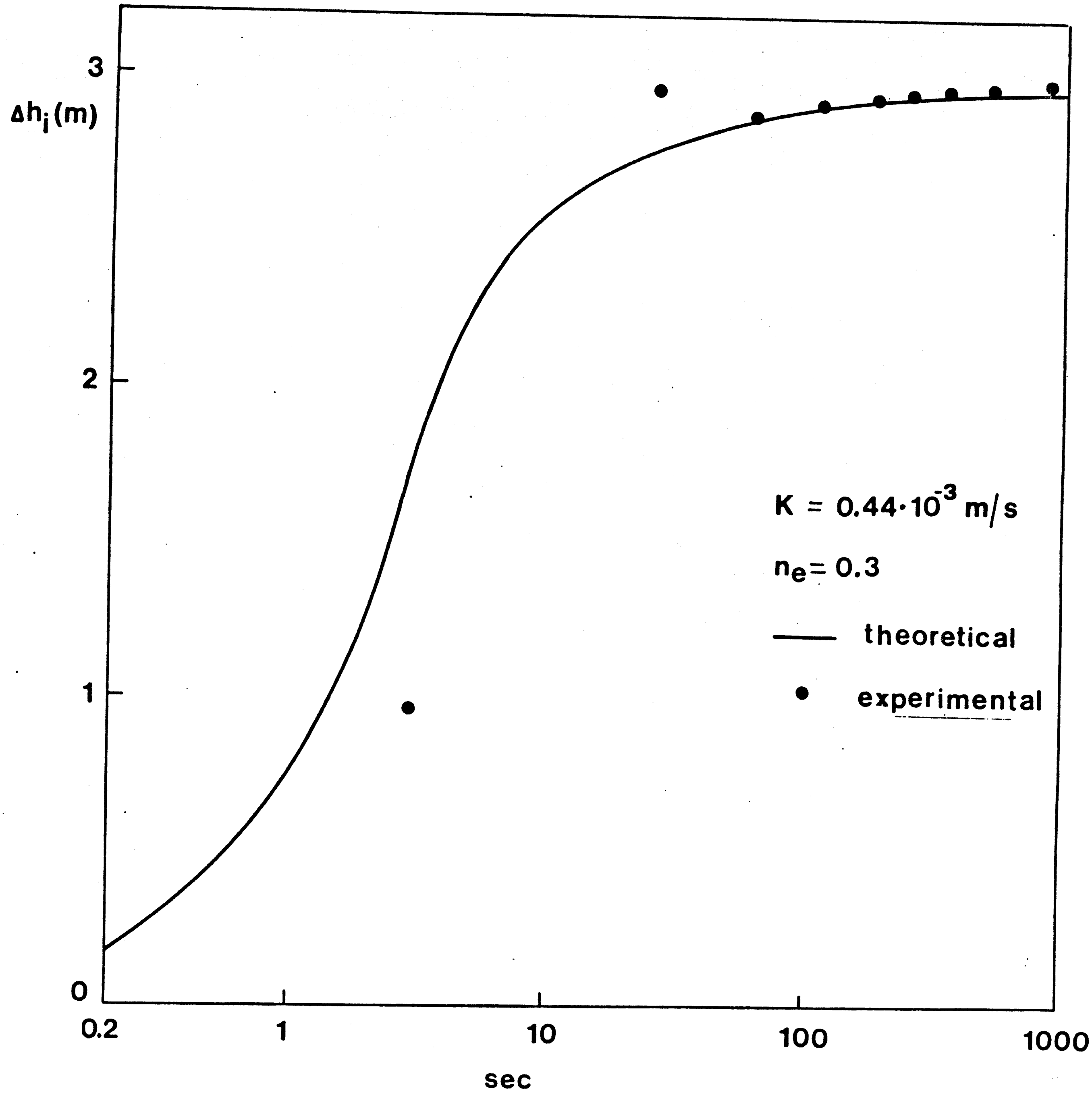

Figure 8. Match between the theoretical outcome from the model and the drawdown recorded in the CORIM test hole.

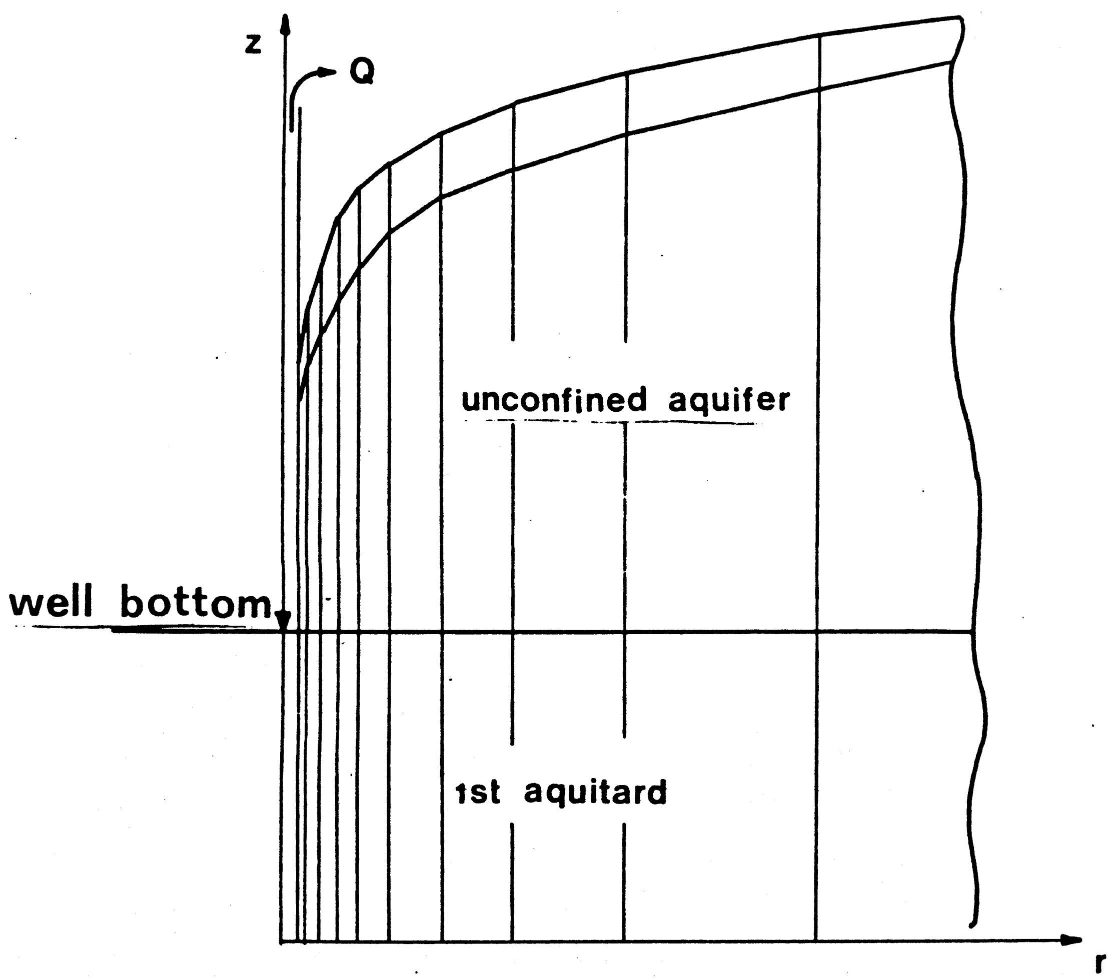

Figure 9. Scheme of the two-layer system as used by the model to check a possible leakage from the 1st artesian aquifer.

RESULTS

In this section we provide a brief account of the results of our simulation. A wider analysis is given in a report currently in preparation.

An important response is the hydrologic balance of the system when the wells tapping the Crescentino unconfined aquifer are pumped at their maximum rates. It has been proven (Hantush 1962, Kirkham 1964) that a success of the Dupuit theory is the correct prediction for the discharge from gravity wells. The bi-dimensional horizontal model is therefore suited to give an estimate of the water supply obtainable from the active existing wells M and F. The flow region has been covered with a rectangular mesh using variable nodal spacing. The mesh is shown in Figure 10 where the location of the pumping wells is indicated as well. The boundary in the N-E direction is set at a large distance from the pumping sites. The linear system of eqs. (4) has been solved over the selected network.

The declared consumption from wells F does not exceed 0.5 m^3/s. The first question to be answered is: what is the maximum theoretical withdrawal from all of the wells M? Predictions are made with the assumption that the wells M are fully penetrating, totally screened and with negligible well losses. The hole diameters are set equal to 0.6 m. Since $W_{i,j}$ are not known a priori in the wells M, some attempts proved necessary to find the final solution. The trials were stopped when h on the well wall obtained by interpolation between the values taken by h in the well and in the nearest nodes was approximately zero.

Table 2 gives the largest withdrawals from wells M when wells F are either shut down (column A) or active (column B). If discharge from wells F equates its declared value, the overall supply from wells M is reduced by 7%. A prospective future demand of about 1 m^3/s has been satisfied by locating 5 new wells at a distance of 300 m from Canale Cavour (identified in Figure 10 by N). Their diameter is 0.6 m again. If the wells F are not operative, simultaneous pumpage from wells M and N exceeds 2 m^3/s (column C). Finally if wells F

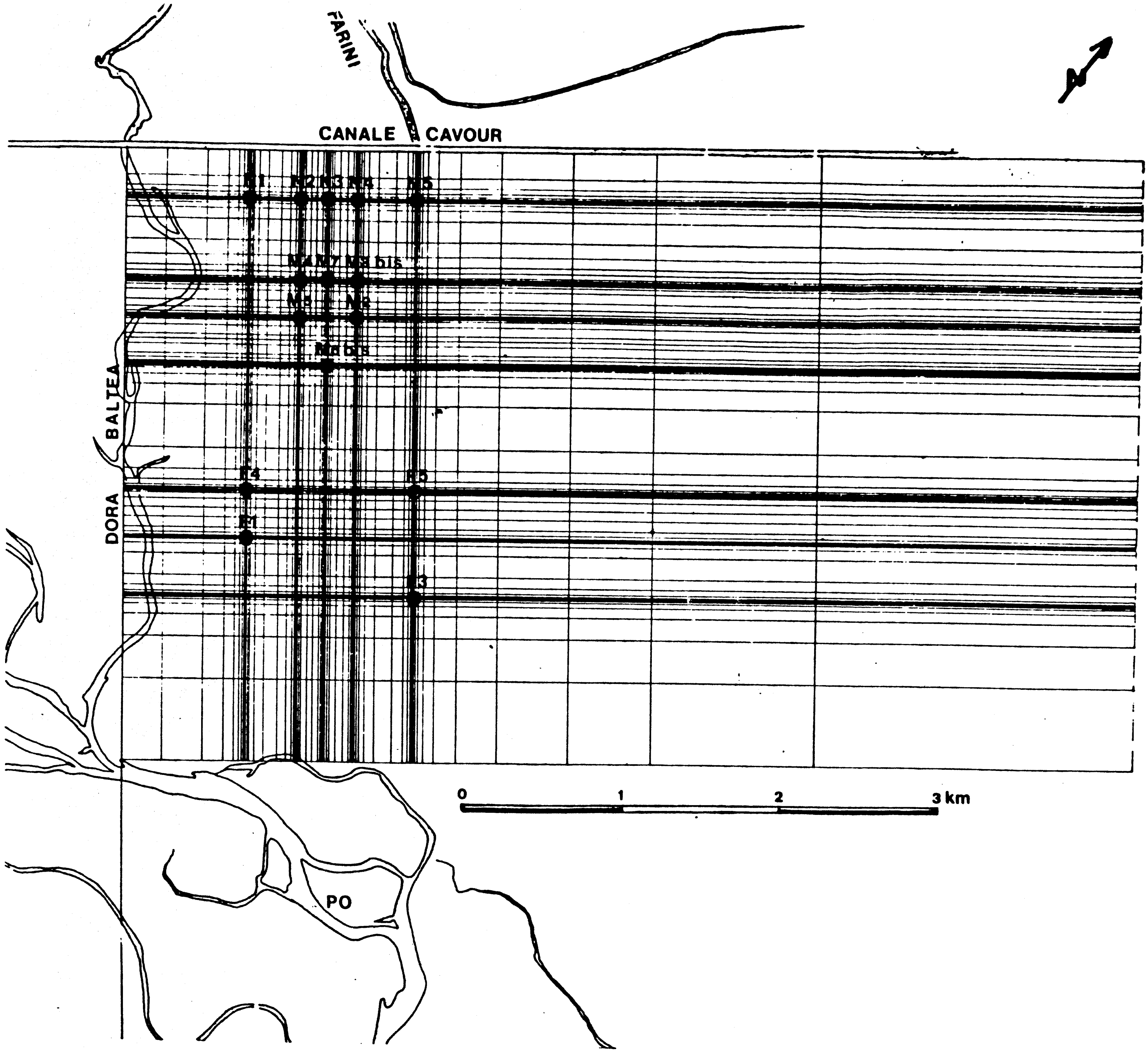

Figure 10. Network used to estimate the maximum pumping rate from wells M,F and N.

Table 1. Drawdowns recorded in the CORIM borehole and in the piezometers during the pumping test.

Time	Δh (m)							Q (l/s)
	test hole	P1	P2	P3	P4	P5	P6	
0.	0.	0.	0.	0.	0.	0.	0.	
3"	0.96							
25"	2.96							
1'	2.87							
2'	2.92							
3'	2.94							
4'	2.95							
5'	2.96							
8'	2.97							
11'	2.97							
14'	2.99							
17'	3.00							
20'	3.00	0.	0.01	0.	0.01	0.	0.	
25'	3.01							24.7
30'	3.01							
40'	3.01	0.	0.01	0.	0.01	0.	0.01	
60'	3.01	0.	0.02	0.	0.01	0.	0.01	
90'	3.02	0.	0.02	0.	0.01	0.	0.01	24.6
120'	3.02	0.	0.02	0.01	0.02	0.01	0.01	
4h	3.02	0.01	0.02	0.01	0.02	0.01	0.01	24.6
5h	3.02	0.02	0.02	0.01	0.02	0.01	0.02	
6h	3.01							25.
7h	3.01	0.02	0.02	0.01	0.02	0.01	0.02	24.6
30h	3.01							24.6
32h	3.02	0.03	0.05	0.04	0.05	0.03	0.05	

Table 2. Maximum pumping rate from wells M and N under various pumping plans.

Well	Q (l/s) A	B	C	D
M4	186	179	134	130
M7	159	151	108	105
M3 bis	181	174	132	127
M5	172	160	143	133
M2	166	152	136	124
M6 bis	202	177	184	158
F1	-	125	-	125
F3	-	125	-	125
F4	-	125	-	125
F5	-	125	-	125
N1	-	-	295	225
N2	-	-	224	218
N3	-	-	209	204
N4	-	-	225	219
N5	-	-	294	280
Total	1066	1493	2084	2483
Total M	1066	993	837	777
Total F	--	500	--	500
Total N	--	--	1247	1206

are active, the respective potential consumptions are shown in column D. Comparison between column C and D indicates that the discharge from wells F affects the water supply from wells M and N by only 5%.

It may be concluded that: 1) a pumping of 0.5 m^3/s from area F has a modest influence on the water supply obtainable from area M; 2) drilling of 5 new wells in the position shown in Figure 10 provides an increase of the total withdrawal of more than 1 m^3/s.

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